FUZZY GOAL PROGRAMMING PROCEDURE TO BI-LEVEL MULTI-OBJECTIVE PROGRAMMING PROBLEMS

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ABSTRACT:

In this paper we will apply fuzzy goal programming (FGP) procedure to solve a Bi-level Multi-objective programming problem. In the proposed approach, the individual optimal decision of each of the decision makers (DMs) located at different hierarchical levels is determined first. In the decision process, the fuzzy goal-programming solution approach is used for achieving the highest degree (utility) of each of the defined membership goals to the extent possible by minimizing their under deviational variables and thereby obtaining the most satisfactory solution for all the decision makers in the decision making environment. The main advantage of the proposed FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process.

Key words: Bi-level programming problems, Fuzzy goal programming, Fuzzy membership functions, satisfactory solution.

[1] INTRODUCTION

In this paper we deal with the Bi-level Multi-objective programming problem (BLMOPP) with the essentially co-operative decision makers (DMs) and propose an algorithm to solve Bi-level Multi-objective programming problem (BLMOPP) via fuzzy goal programming.

Multi-level programming is a powerful technique for solving hierarchical decision-making problems. Multi-level optimization plays an important role in engineering design, management, and decision making in general. Ultimately, a designer or decision maker needs to make tradeoffs between disparate and conflicting design objectives. The field of multi-level optimization defines the art and science of making such decisions. The prevailing approach for address this decision-making task is to solve an
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optimization problem, which yields a candidate solution. Multi-level programming problem can be defined as a p-person, non-zero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure.

Hierarchical optimization or multi-level programming techniques are extension of Stackelberg games for solving decentralized planning problem with multiple DMs in a hierarchical organization. The Stackelberg solution has been employed as a solution concept to bi-level programming problems (BLPPs), and a considerable number of algorithms for obtaining the solution have been employed. A tri-level programming problem (TLPP) and bi-level programming problem is a special case of multi-level programming problem (MLPP).


Hierarchical optimization or multilevel programming problems (MLPPs) have the following common characteristics: interactive decision making units exist within predominantly hierarchical structures; the execution of decision is sequential from higher level to lower level; each decision making unit independently controls a set of decision variables and is interested in maximizing its own objective but is affected by the reaction of lower level decision makers (DMs). Due to their dissatisfaction with the decision of the higher level DMs, decision deadlock arises frequently in the decision-making situation. Some important existing solution approaches such as the extreme point search, the procedure based on the Karush-Kuhn Tucker condition, and the decent method (Anandilingam and Apprey 1991; Biswas and Pal 2005; Bellmann 1957; Charnes and Cooper 1962[2]; Craven and Mond 1975; Lai 1996[3]) are effective only for solving simple types of multilevel programming problems. Initially, fuzzy approach was used to handle multiobjective optimization problems (Chakraborty and Gupta 2002; Jimenez and Bilbas 2009). Lai and Hwang (1993) at first developed an effective fuzzy approach using the concept of tolerance membership functions for solving MLPPs in 1996. Shih et al. (1996) extended Lai’s concept using a non-compensatory maximum-minimum aggregation operator for solving MLPPs. Shih and Lee (2000)[15] further extended Lai’s concept by introducing the compensatory fuzzy operator for solving MLPPs. Sinha (2003a,b) studied alternative MLP techniques based on fuzzy mathematical programming (FMP). The basic concept of these fuzzy approaches is the same, and evaluation of the problem again and again by redefining the elicited membership values is essentially needed in the solution search process to obtain a satisfactory solution. So, computational load is also inherently involved in the fuzzy approaches developed so far. In the FMP techniques of Sinha (2003a,b), the last (lower) level is the most important, and the decision of the lowest level remains either unchanged or closest to individual best decisions, which leads to the paradox that the decision power of the lowest level DM dominates the higher level DM. To overcome such difficulties, the fuzzy goal programming (FGP) approach to multi-decision-making problems was introduced by Mohamed (1997) which is extended by Pramanik and Roy (2007)[12] to solve MLPPs. Baky (2009) used fuzzy goal programming to solve decentralized bi-level multi-objective programming problems. Chang (2009) suggested goal programming approach for fuzzy multi-objective fractional programming problems. Recently, Pal and Gupta (2009) studied a genetic algorithm to fuzzy goal programming formulation of fractional multi-objective decision-making problems.

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In this paper we deal with the Bi-level Multi-objective programming problem (BLMOPP) with the essentially cooperative DMs and propose an algorithm to solve bi-level Multi-objective programming problem (BLMOPP) via Fuzzy Goal Programming (FGP). In this paper, the FGP approach is extended to solve the proposed BLMOPP. In the model formulation of the problem, the fuzzy goal levels of the objectives as well as the decision vectors controlled by the upper-level DMs are defined first. The fuzzy goals are then characterized by the associated membership functions. In the solution process, first the membership functions are defined as flexible membership goals by introducing under- and over-deviational variables and assigning highest membership value (unity) as aspiration level to each of them. Then in the achievement function, minimization of the under-deviational variables for achieving the membership goals to the highest degree to the extent possible on the basis of their importance is taken into account.
[3] BI-LEVEL MULTI-OBJECTIVE PROGRAMMING PROBLEMS

A bi-level multi-objective programming problem (BLMOPP) consists of two levels, namely, the first level and the second level and each has linear or quadratic fractional objective function. A multi-level programming problem (MLPP) can be defined as a p-person, nonzero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure. When \( p = 2 \), we call the system a bi-level programming problem.

For instance, by adopting a criterion with respect to finance or corporate planning as an objective function at the upper level and employing a criterion regarding production planning as an objective function at the lower level, a bi-level linear or non-linear fractional programming problem can be formulated for hierarchical decision problems in firms.

[4] A BI-LEVEL MULTI-OBJECTIVE PROGRAMMING PROBLEM IS MATHEMATICALLY FORMULATED AS:

\[
\begin{align*}
\max_{x_1} Z_1(x_1, x_2) &= \max_{x_1} (z_{11}, z_{12}, \ldots, z_{1m}) \\
\max_{x_2} Z_2(x_1, x_2) &= \max_{x_2} (z_{21}, z_{22}, \ldots, z_{2n})
\end{align*}
\]

Where \( x_2 \) solves

\[
\max_{x_2} Z_2(x_1, x_2) = \max_{x_2} (z_{21}, z_{22}, \ldots, z_{2n})
\]

Subject to

\[
A_1x_1 + A_2x_2 \leq b, x_1, x_2 \geq 0
\]

Where objective functions \( z_j(x_1, x_2), (i = 1, 2) \) are represented by a quadratic fractional function

\[
z_j(x_1, x_2) = \frac{Q_{ij} X^2 + C_{ij} X + \alpha_{ij}}{R_{ij} X^2 + D_{ij} X + \beta_{ij}}
\]

\( j=1, 2, \ldots, m_1, i=1 \) for ULDM objective functions,

\( j=1, 2, \ldots, m_2, i=2 \) for LLDM objective functions,

and where

(i) \( x_1 = (x_1^1, x_1^2, \ldots, x_1^{m_1}) \), \( x_2 = (x_2^1, x_2^2, \ldots, x_2^{m_2}) \),

(ii) \( m_1, m_2 \) be the number of first and second level objective functions.

(iii) \( Q_{ij}, R_{ij}, C_{ij}, D_{ij} \in R^+ \), \( R_{ij} X^2 + D_{ij} X + \beta_{ij} > 0 \)

(iv) \( \alpha_{ij}, \beta_{ij} \) are constant.
[5] FUZZY GOAL PROGRAMMING APPROACH TO BLMOPP

To formulate the fuzzy programming model of BLMOPP under consideration, the objective functions $z_j$ and decision vectors are required to be transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then they are characterized by their membership functions by defining the tolerance limits for achievement of the aspired levels of the goals.

[6] CONSTRUCTION OF MEMBERSHIP FUNCTION

Since the DMs of both the levels are interested in optimizing their individual benefit over some feasible region, the optimal solution of each of them calculated in isolation would be the aspiration levels of their associated fuzzy goals.

Let $(x_1^1, x_2^1, z_1^{max})$ be the solution for the upper level problem and $(x_1^2, x_2^2, z_2^{max})$ be the solution for the lower level problem. Then the decision maker at upper level gives some tolerance to the decision vector and the objective function. Tolerance on the decision vector of upper level enables the decision maker at lower level to search for his optimum in a wider feasible range. By giving some tolerance to his objective function the decision maker at upper level directs the decision maker at lower level to search for his solutions in the right direction.

We include the membership functions for the fuzzy goals of the decision variables controlled by first-level decision maker $(x_1^1, x_2^1, ..., x_n^1)$ in the proposed model in this article. To build these membership functions, the optimal solution $x^* = (x_1^*, x_2^*)$ of the upper-level MOQFP problem should be determined first.

Let $t_k^L$ and $t_k^R$, $k = 1, 2, ..., n_1$ be the maximum negative and positive tolerance values on the decision vector considered by the ULDM. The tolerance $t_k^L$ and $t_k^R$, $k = 1, 2, ..., n_1$ are not necessarily same. The linear membership functions for the decision vector $x_i = (x_1^i, x_2^i, ..., x_n^i)$, can be formulated:

$$\mu_{x_i^K}(x^K_i) = \frac{[x_i^K - (x_i^{K*} - t_k^L)]}{t_k^L}, x_i^{K*} - t_k^L \leq x_i^K \leq x_i^{K*}$$

$$\mu_{x_i^K}(x^K_i) = \frac{[(x_i^{K*} + t_k^R) - x_i^K]}{t_k^R}, x_i^{K*} \leq x_i^K \leq x_i^{K*} + t_k^R$$

0, otherwise

Then, the fuzzy goals of the decision makers objective functions at both levels and the vector of fuzzy goals of the decision variables controlled by first-level decision maker appear as

$$Z_{ij} \leq g_{ij}, i = 1, 2, j = 1, 2, ..., m_i, x_i = x_i^*$$

Then, membership functions $\mu_{Z_{ij}}$ for the $ij^{th}$ fuzzy goal can be formulated as:

$$\mu_{Z_{ij}}$$
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1, \( f_{ij} \leq g_{ij} \)

\[ \mu_{z_{ij}}(x) = \frac{[u_{ij} - Z_{ij}(x_1, x_2)] - g_{ij}}{u_{ij} - g_{ij}}, \quad g_{ij} \leq f_{ij} \leq u_{ij} \]

0, \( f_{ij}(x) \geq u_{ij} \)

(3)

Now, in the decision making situation, the aim of each DM is to achieve the highest membership value of the associated fuzzy goal. But in actual practice, achievement of all membership values to the highest degree is not possible due to limitation of the resources. In such a case, the FGP solution technique, as a robust and most flexible technique for solving multi-objective decision analysis, is used to solve the problem of achieving the highest degree (unity) of the defined membership functions and thereby obtaining the most satisfactory decision.

[7] FUZZY GOAL PROGRAMMING SOLUTION APPROACH

In decision making situation, the aim of each DM is to achieve highest membership value (unity) of the associated fuzzy goal in order to obtain the absolute satisfactory solution. However, in real practice, achievement of all membership values to the highest degree (unity) is not possible due to conflicting objectives. Therefore, decision policy for minimizing the regrets of the DMs for all the levels should be taken into consideration. Hence, each DM should try to maximize his or her membership function by making them as close as possible to unity by minimizing its negative deviational variables. Therefore, in effect, we are simultaneously optimizing all the objective functions. So, for the defined membership functions in (2),(3) and (4), the flexible membership goals having the aspired level unity can be represented as:

\[ [u_{ij} - Z_{ij}(x_1, x_2)]/[u_{ij} - z_{ij}] + d^-_{ij} - d^+_{ij} = 1, \quad i = 1, 2, j = 1, 2, ..., m_i \]

\[ [x^k_1 - (x^k_1 - t^L_k)]/t^L_k + d^L_1 - d^{L+}_1 = 1, k = 1, 2, ..., n_1 \]

\[ [(x^k_1 + t^R_k) - x^k_1]/t^R_k + d^R_k - d^{R+}_k = 1, k = 1, 2, ..., n_1 \]

Where \( d^-_{ij}, d^+_{ij} \geq 0 \) , \hspace{1cm} \text{------------------------ (a)}

\( d^-_1, d^+_1 \geq 0 \), \hspace{1cm} \( d^L_k, d^{L+}_k \geq 0 \), \hspace{1cm} \text{------------------------ (b)}

Here (a) represents the over and under deviational variables and (b) represents the vectors of over and under deviational variables associated with the respective goals. \( I \) is the column vector having all components equal to 1.
The FGP approach to multiobjective programming problems presented by Mohamed is extended here to formulate the FGP approach to bi-level multi-objective linear fractional programming. Therefore, considering the goal achievement problem of the goals at the same priority level, the equivalent fuzzy bilevel multiobjective linear fractional goal programming model of the problem can be presented as:

\[
\text{Minimize } Z = \sum_{j=1}^{m} W_{ij}^+ d_{ij}^+ + \sum_{k=1}^{n} [W_k^L (d_k^{L+} + d_k^{L-}) + W_k^R (d_k^{R+} + d_k^{R-})] + \sum_{j=1}^{m} W_{ij}^- d_{ij}^-
\]

Subject to

\[
[u_{ij} - Z_{ij} (x_1, x_2)]/[u_{ij} - z_{ij}] + d_{ij}^- - d_{ij}^+ = 1, \quad i = 1, 2, j = 1, 2,..., m
\]

\[
[x_i^k - (x_i^{R+} - t_i^k)]/t_i^L + d_i^{L-} - d_i^{L+} = 1, k = 1, 2,..., n_i
\]

\[
[(x_i^{L+} + t_i^{R+}) - x_i^{R+}] / t_i^R + d_i^{R-} - d_i^{R+} = 1, k = 1, 2,..., n_i
\]

\[
d_{ij}^-, d_{ij}^+ \geq 0 ,
\]

\[
d_k^{R-}, d_k^{R+} \geq 0,
\]

In the present formulation, numerical weights \( W_{ij}^+ \geq 0 \) and the vectors of the numerical weight \( w_k^L, w_k^R \geq 0 \) represents the relative importance of the goals for achieving their aspired levels. The above FGP model provides the most satisfactory decision by achieving the aspired levels of the membership goals to the extent possible in the decision making environment.

[8] CONCLUSION:

An effort has been made to solve BLMOPP based on fuzzy set theory and goal programming approach. The main advantage of the FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process. The proposed approach can be extended to solve fuzzy multi-objective MLPPs. An extension of the approach for fuzzy multi-level decentralized is one of the current research problems. However, it is hoped that proposed approach can contribute to future study in the field of practical hierarchical decision making problems.

[9] NUMERICAL EXAMPLE:

In this section we present numerical example to demonstrate the solution procedures by proposed approach to solve bi-level quadratic fractional programming problem (BLQFPP).
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following example considered by Mishra [7] is again used to demonstrate the solution procedures and clarify the effectiveness of the proposed approach:

Consider the following BLQFPP (Special case of BLMOPP)

$$\max_{x_1} z_1 = \frac{10x_1^2 + 15x_2^2 + 5}{x_1^2 + 2x_2^2 + 1}$$

where $x_2$ solves

$$\max_{x_2} z_2 = \frac{25x_1^2 + 9x_2^2}{2x_1^2 + x_2^2 + 1}$$

subject to

$$4x_1 - 5x_2 \leq 15; \ 3x_1 - x_2 \leq 21; \ 2x_1 + x_2 \leq 27; \ 3x_1 + 4x_2 \leq 45; \ x_1 + 3x_2 \leq 30;$$

$$x_1 \geq 0, \ x_2 \geq 0.$$ 

Solution by proposed approach:

The satisfactory solution is as follows:

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.34</td>
<td>1.27</td>
<td>9.6</td>
<td>12.20</td>
</tr>
</tbody>
</table>

Realized degree of satisfaction is:

$$\mu_{x_1} = \left[ x_1 - (x_1^u - p_1) \right] / p_1 = 0.96;$$

$$\mu_{z_1} = \left( z_1 - z_1^L \right) / \left( z_1^U - z_1^L \right) = 0.7$$

$$\mu_{z_2} = \left( z_2 - z_2^L \right) / \left( z_2^U - z_2^L \right) = 0.7;$$

The solution to the problem is feasible. Hence, (5.34, 1.27) is the optimal solution of the given BLQFPP.

Note: All solution of the problem are obtained by using the software Lingo, version 6.0.
REFERENCES