QUADRATIC FRACTIONAL MULTILEVEL PROGRAMMING PROBLEM BASED ON FUZZY GOAL PROGRAMMING APPROACH

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ABSTRACT:

In this paper we will apply fuzzy goal programming (FGP) approach to solve a Multi-level quadratic fractional programming problem. In the proposed approach, the individual optimal decision of each of the decision makers (DMs) located at different hierarchical levels is determined first. In the decision process, the fuzzy goal-programming solution approach is used for achieving the highest degree (utility) of each of the defined membership goals to the extent possible by minimizing their under deviational variables and thereby obtaining the most satisfactory solution for all the decision makers in the decision making environment.

The main advantage of the proposed FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process.

Key words: Bi-level programming problems, Fuzzy goal programming, Fuzzy membership functions, satisfactory solution.

[1] INTRODUCTION

In this paper we deal with the quadratic fractional multi-level programming problem (QFMLPP) with the essentially co-operative decision makers (DMs) and propose an algorithm to solve multi-level quadratic fractional programming problem via fuzzy goal programming.

Multi-level programming is a powerful technique for solving hierarchical decision-making problems. Multi-level optimization plays an important role in engineering design, management, and decision making in general. Ultimately, a designer or decision maker needs to make tradeoffs between disparate and conflicting design objectives. The field of multi-level optimization defines the art and science of making such decisions. The prevailing approach for addressing this decision-making task is to solve an optimization problem, which yields a candidate solution. Multi-level programming problem can be defined as a p-person,
non-zero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure.

Hierarchical optimization or multi-level programming techniques are extension of Stackelberg games for solving decentralized planning problem with multiple DMs in a hierarchical organization. The Stackelberg solution has been employed as a solution concept to bi-level programming problems (BLPPs), and a considerable number of algorithms for obtaining the solution have been employed. A tri-level programming problem (TLPP) and bi-level programming problem is a special case of multi-level programming problem (MLPP).

Hierarchical optimization or multilevel programming problems (MLPPs) have the following common characteristics: interactive decision making units exist within predominantly hierarchical structures; the execution of decision is sequential from higher level to lower level; each decision making unit independently controls a set of decision variables and is interested in maximizing its own objective but is affected by the reaction of lower level decision makers (DMs). Due to their dissatisfaction with the decision of the higher level DMs, decision deadlock arises frequently in the decision-making situation. Some important existing solution approaches such as the extreme point search, the procedure based on the Karush-Kuhn Tucke condition, and the decent method (Anandilingam[1]1988;Anandilingam and Apprey 1991; Biswas and Pal 2005;Bellmann 1957; Charnes and Cooper[2] 1962; Craven and Mond 1975; Lai [3]1996) are effective only for solving simple types of multilevel programming problems. Initially, fuzzy approach was used to handle multiobjective optimization problems (Chakraborty and Gupta 2002; Jimenez and Bilbas 2009). Lai and Hwang (1993) at first developed an effective fuzzy approach using the concept of tolerance membership functions for solving MLPPs in 1996. Shih et al. (1996) extended Lai’s concept using a non-compensatory maximum-minimum aggregation operator for solving MLPPs. Shih and Lee[15] (2000) further extended Lai’s concept by introducing the compensatory fuzzy operator for solving MLPPs. Sinha (2003a,b) studied alternative MLP techniques based on fuzzy mathematical programming (FMP). The basic concept of these fuzzy approaches is the same, and evaluation of the problem again and again by redefining the elicited membership values is essentially needed in the solution search process to obtain a satisfactory solution. So, computational load is also inherently involved in the fuzzy approaches developed so far. In the FMP techniques of Sinha (2003a,b), the last (lower) level is the most important, and the decision of the lowest level remains either unchanged or closest to individual best decisions, which leads to the paradox that the decision power of the lowest level DM dominates the higher level DM. To overcome such difficulties, the fuzzy goal programming (FGP) approach to multidecision-making problems was introduced by Mohamed (1997) which is extended by Pramanik and Roy[12] (2007) to solve MLPPs. Baky (2009) used fuzzy goal programming to solve decentralized bilevel multiobjective programming problems. Chang (2009) suggested goal programming approach for fuzzy multiobjective fractional programming problems. Recently, Pal and Gupta (2009) studied a genetic algorithm to fuzzy goal programming formulation of fractional multiobjective decision-making problems. S.Pramanik, and P. Dey [11] in 2011 has introduced Multi-Objective Quadratic Programming Problem Based on Fuzzy Goal Programming. In the proposed approach, after formulating the quadratic membership functions it is linearized into equivalent linear membership functions at the best solution point by using first order Taylor polynomial series. Then fuzzy goal programming technique was used for solving the problem by minimizing only the negative deviational variables. The same concept was extended by S.
Pramanik and P. Dey [10] to solve BLLFPP in 2011 and by S. Pramanik and D. Banerjee to solve chance constrained quadratic bi-level programming problem in 2012. M. Saraj and S. Sadeghi [16] in 2012 has proposed a fuzzy goal programming method for obtaining a satisfactory solution to a bi-level multi-objective absolute value fractional programming (BLMO-A-FP) problems. The method of variable change on the under- and over-deviation variables of the membership goals associated with the fuzzy goals of the model was introduced to solve the problem efficiently by using linear goal programming methodology. K. Lachhwani [4] in 2012 has introduced fuzzy goal programming (FGP) procedure for multi-objective quadratic programming problem. In the FGP model formulation, firstly the objectives are transformed into fuzzy goals (membership functions) Then achievement of the highest membership value of each of fuzzy goals was formulated by minimizing the negative deviational variables, again in the same year K. Lachhwani and M. Poonia [4] have given a procedure for solving multilevel fractional programming problems in a large hierarchical decentralized organization using fuzzy goal programming approach. Biswas, A.; Bose, K.; Dewan, S. in 2013 has studied a fuzzy goal programming approach for quadratic multiobjective bilevel programming under fuzzy environment. In the year of 2013 P. Dey, S. Pramanik and Bibhas C. Giri has given an extended work on Bi-level multi-objective linear fractional programming problems. M. Kumar, B. Baran Pal in 2013 presents a fuzzy goal programming (FGP) procedure for solving multilevel programming problems (MLPPs) having chance constraints in large hierarchical decision organizations.

In this paper we deal with the multi-level quadratic fractional programming problem (MLQFPP) with the essentially cooperative DMs and propose an algorithm to solve multi-level quadratic fractional programming problem via. Fuzzy Goal Programming (GP). It is a simple method to apply in the multi-level systems compared to the other transformation method. In this paper, the FGP approach in is extended to solve the proposed MLQFPP. A usual quadratic fractional programming problem is a special case of a non-linear programming problem, but it can be transformed into a quadratic programming problem by using the transformation [7]. In the model formulation of the problem, the fuzzy goal levels of the objectives as well as the decision vectors controlled by the upper-level DMs are defined first. The fuzzy goals are then characterized by the associated membership functions. In the solution process, first the membership functions are defined as flexible membership goals by introducing under- and over-deviational variables and assigning highest membership value (unity) as aspiration level to each of them. Then in the achievement function, minimization of the under-deviational variables for achieving the membership goals to the highest degree to the extent possible on the basis of their importance is taken into account.

[2] A MULTI-LEVEL QUADRATIC FRACTIONAL PROGRAMMING PROBLEM IS MATHEMATICALLY FORMULATED AS:

We consider a T-level quadratic fractional programming problem of maximization –type objectives function at each level. Mathematically, we can state it as follows:

$$\max Z_i(x) = \frac{Q_{i1}x_1^2 + Q_{i2}x_2^2 + \ldots + Q_{iT}x_T^2 + c_{i1}x_1 + c_{i2}x_2 + \ldots + c_{iT}x_T + \alpha_i}{R_{i1}x_1^2 + R_{i2}x_2^2 + \ldots + R_{iT}x_T^2 + d_{i1}x_1 + d_{i2}x_2 + \ldots + d_{iT}x_T + \beta_i}$$
Maximize $Z_T(x) = \frac{Q_{t1}x_1^2 + Q_{t2}x_2^2 + \ldots + Q_{tT}x_T^2 + c_{t1}x_1 + c_{t2}x_2 + \ldots + c_{tT}x_T + \alpha_T}{R_{t1}x_1^2 + R_{t2}x_2^2 + \ldots + R_{tT}x_T^2 + d_{t1}x_1 + d_{t2}x_2 + \ldots + d_{tT}x_T + \beta_T}$

Subject to

$A_i x_1 + A_{i2} x_2 + \ldots + A_{iT} x_T (\leq, =, \geq) b_i, \forall i = 1, 2, \ldots, m, x_i \geq 0, x_2 \geq 0, \ldots, x_T \geq 0$ (1)

[3] FUZZY GOAL PROGRAMMING APPROACH TO MLQFPP

To formulate the fuzzy programming model of MLQFPP under consideration, the objective functions $z_1, z_2, \ldots, z_T$ and decision vectors $x_t, (t = 1, 2, \ldots, T - 1)$ are required to be transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then they are to be characterized by their membership functions by defining tolerance limits for the achievement of the aspired levels of then corresponding fuzzy goals.

[4] CONSTRUCTION OF MEMBERSHIP FUNCTION OF MLQFPP

In the decision-making context, each DM is interested in maximizing his or her own objective function; the optimal solution of each DM when calculated in isolation would be considered as the best solution, and the associated objective values can be considered as the aspiration level of the corresponding fuzzy goal. Let $x^{''}_t$ be the best solution of the $t$-th level DM. It is quite natural that objective values which are equal to or larger than $z^{''}_t = \max z_t(x), t = 1, 2, \ldots, T$ should be absolutely satisfactory to the $t$-th level DM. If the individual best solutions $x^{''}_t, t = 1, 2, \ldots, T - 1$, are the same, then a satisfactory optimal solution of the system is reached. However, this rarely happens due to the conflicting nature of the objectives. To obtain a satisfactory solution, the higher DM should give some tolerance (relaxation), and the relaxation of the decision of the higher level DM depends on the needs, desires, and practical situations in the decision-making situation. Then, the fuzzy goals take the form

$z_t(x) \geq z_t(x^{''}), t = 1, 2, \ldots, T$

And $x_t \approx x^{''}_t, t = 1, 2, \ldots, T - 1$

Here, $x_t, (t = 1, 2, \ldots, T)$ are assumed to be the main decision vectors.

$z^{''}_t = z_t(x^{''}) = \max z_t(x), t = 1, 2, \ldots, T$ give upper tolerance limit or aspired level of achievement for the $t$-th objective function.

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where \( z_i^L = \min z_i(x), t = 1,2,...,T \) gives the lower tolerance limit or lowest acceptable level of achievement for the \( t \)-th objective function.

Then the membership function for the defined fuzzy goal appears as follows:

\[
1, z_i(x) \geq z_i^u \\
\mu_i[z_i(x)] = [z_i(x) - z_i^L]/[z_i^u - z_i^L], z_i^L \leq z_i(x) \leq z_i^u \\
0, z_i(x) \leq z_i^L, \forall t = 1,2,...,T
\]  \( \text{(2)} \)

Here, linear membership functions are more suitable than nonlinear functions as less computational difficulties arise in models due to it. Let \( p_i^- \) and \( p_i^+ \) (\( t = 1,2,...,T - 1 \)) be the negative and positive tolerance values on decision vectors \( x_i \) considered by the \( t \)-th level DM. This is a triangular fuzzy number.

Then, the linear membership functions for decision vectors \( x_i \) can be formulated as follows:

\[
\mu_i^u(x_i) = [x_i - (x_i^u - p_i^-)]/p_i^- , (x_i^u - p_i^-) \leq x_i \leq x_i^u \\
[(x_i^u + p_i^+) - x_i]/p_i^+, x_i^u \leq x_i \leq x_i^u + p_i^+ \]  \( \text{(3)} \)

\[0, otherwise \forall t = 1,2,...T - 1\]

Now, in the decision making situation, the aim of each DM is to achieve the highest membership value of the associated fuzzy goal. But in actual practice, achievement of all membership values to the highest degree is not possible due to limitation of the resources. In such a case, the FGP solution technique, as a robust and most flexible technique for solving multi-objective decision analysis, is used to solve the problem of achieving the highest degree (unity) of the defined membership functions and thereby obtaining the most satisfactory decision.

[5] FUZZY GOAL PROGRAMMING SOLUTION APPROACH

In decision making situation, the aim of each DM is to achieve highest membership value (unity) of the associated fuzzy goal in order to obtain the absolute satisfactory solution.
However, in real practice, achievement of all membership values to the highest degree (unity) is not possible due to conflicting objectives. Therefore, decision policy for minimizing the regrets of the DMs for all the levels should be taken into consideration. Hence, each DM should try to maximize his or her membership function by making them as close as possible to unity by minimizing its negative deviational variables. Therefore, in effect, we are simultaneously optimizing all the objective functions. So, for the defined membership functions in (2) and (3) the flexible membership goals having the aspired level unity can be represented as:

$$\mu_i[z_i(x)] + D^-_{tl} - D^+_{tl} = 1; t = 1,2,\ldots, T$$

(4)

$$\mu_i(x_t) + D^-_{t2} - D^+_{t2} = 1; t = 1,2,\ldots, (T - 1)$$

(5)

Where, $$D^-_{t1}, D^+_{t1}, D^-_{t2}, D^+_{t2} \geq 0$$

(6)

Here, $$D^-_{t1}$$ is negative deviational variable, and $$D^+_{t1}$$ is positive deviational variable; $$D^-_{t2}, D^+_{t2}$$ represent the vector of negative deviational and positive deviational variables. $$I$$ is the column vector having all components equal to 1. It is to be noted that any over deviation from a fuzzy goal implies the full achievement value. Then the equation (4) and (5) can be written as follows:

$$\mu_i[z_i(x)] + D^-_{t1} \geq 1; t = 1,2,\ldots, T$$

(7)

$$\mu_i(x_t) + D^-_{t2} \geq I; t = 1,2,\ldots, (T - 1)$$

(8)

Here, $$D^-_{t1}$$ is negative deviational variable, and $$D^-_{t2}$$ represent the vector of under deviational variables. $$I$$ is the column vector having all components equal to 1. Now, FGP approach to the problem can be presented as:

Minimize Z = \(\sum_{t=1}^{T} W_{t1} D^-_{t1} + \sum_{t=1}^{T-1} W_{t21} D^-_{t21} + \sum_{t=1}^{T-1} W_{t22} D^-_{t22}\)

(9)

Subject to

$$\left[z_i(x) - z^L_i\right]/\left[z^U_i - z^L_i\right] + D^-_{t1} \geq 1, t = 1,2,\ldots,T$$

$$\left[x_t - (x^a_t - p^-_{t1})\right]/p^-_{t1} + D^-_{t21} \geq I, t = 1,2,\ldots,T - 1$$

$$\left[(x^a_t + p^+_{t1}) - x_t\right]/p^+_{t1} + D^-_{t22} \geq I, t = 1,2,\ldots,T - 1$$
\[ D_{i1}^t \geq 0; t = 1,2,\ldots,T \]
\[ D_{i2}^t \geq 0; t = 1,2,\ldots,(T - 1) \]
\[ A_{i1}x_1 + A_{i2}x_2 + \cdots + A_{iT}x_T (\leq, =, \geq)b_i, \forall i = 1,2,\ldots,m, \]
\[ \text{and } x_i \geq 0, x_2 \geq 0, \cdots, x_T \geq 0 \]  \hspace{1cm} (10)

Here, \( D_{i1}^t \) is negative deviational variable, and \( D_{i21}, D_{i22} \) represent the vector of under deviational variables. \( I \) is the column vector having all components equal to 1.

In the present formulation, numerical weight \( W_{i1}^t (\geq 0) \), and the vectors of the numerical weight \( W_{i21}^-, W_{i22}^- (\geq 0) \), represents the relative importance of the goals for achieving their aspired levels, and they are determined as:

\[ W_{i1}^- = 1/[(z_i^U - z_i^L)] \]
\[ W_{i21}^- = 1/p_i^- \quad \text{and} \quad W_{i22}^- = 1/p_i^+ \]

By solving Equation (10), if the DMs are satisfied with this solution, then a satisfying solution is reached. Otherwise, higher level DMs should provide new tolerance limits for the control variable until a satisfactory solution is reached. In general, considering a set of positive relaxation offered by the higher level DMs, the solution of Equation (10) becomes satisfying for all the level DMs.

[6] CONCLUSION

An effort has been made to solve MLQFPP based on fuzzy set theory and goal programming approach. The main advantage of the FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process. The proposed approach can be extended to solve fuzzy multi-objective MLPPs. An extension of the approach for fuzzy multi-level decentralized is one of the current research problems. However, it is hoped that proposed approach can contribute to future study in the field of practical hierarchical decision making problems.

REFERENCES


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